Accelerator Magnet Design

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References - Acknowledgments

- USPAS course “Superconducting Accelerator Magnets”, Ezio Todesco, Paolo Ferracin, Soren Prestemon
- USPAS Course “Magnetic Systems: Insertion Devices”, Ross Schlueter

Outline

- The magnets of an accelerator
- Some magnetics fundamentals
- Review of magnetic multipoles
  – Definition: Taylor series
  – Inverse problem: how to create multipole fields
    - Iron-dominated (scalar potential)
    - Biot-Savart
- Design and fabrication issues with real accelerator magnets

Layout of an accelerator

- Magnets play key role:
  - Kick beam into accelerator during injection: Kicker magnets
  - Align injected beam with stored beam: Septum magnets
  - Bend beam in circle: bend magnets (dipoles)
  - Focus beam to allow storage (quadrupoles)
  - Compensate for electron energy variation (sextupoles)

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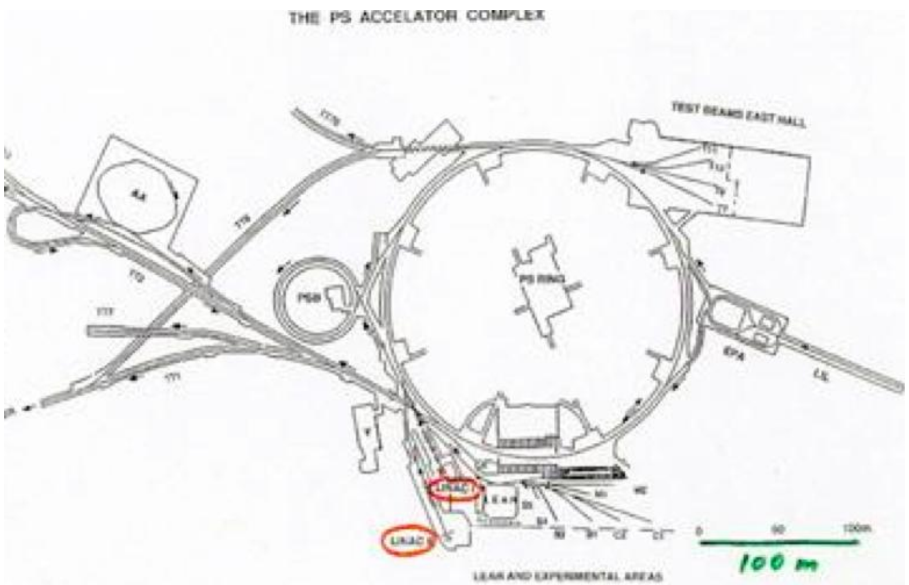
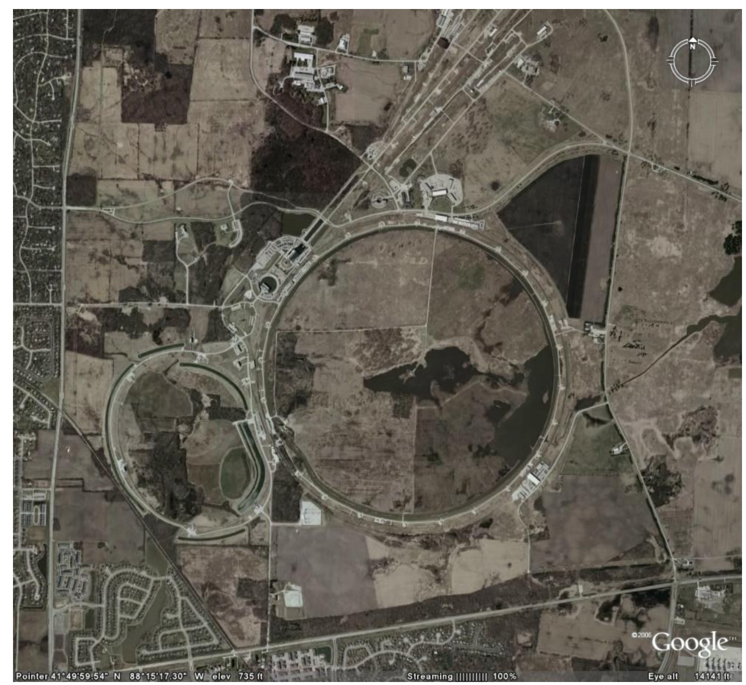




Additional magnet systems

- Correctors
  - Dipoles for field trajectory correction
  - Can be "slow": compensate static or slow-varying drifts
  - Can be fast: allow fast feedback for beam control
- Chicanes
  - Versions of corrector magnets (not used for beam feedback)
  - Used to provide mild steering of beam, e.g. in straights, or for dispersion
- Light-source Wigglers and undulators
  - Used in to produce synchrotron radiation of particular quality
  - Ideally are transparent to beam storage

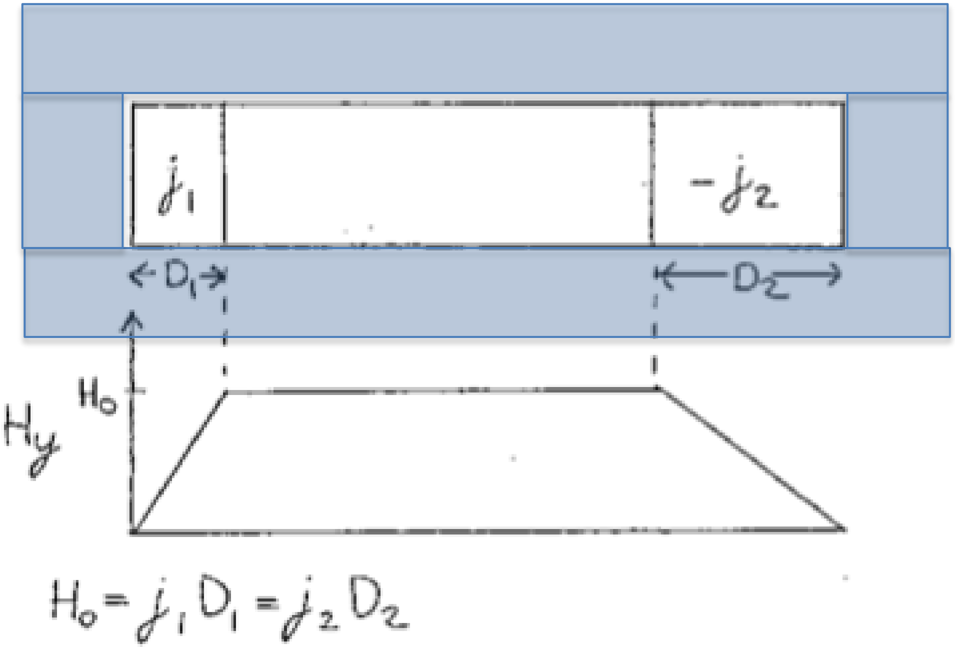
Examples of Accelerator Magnets

- Dipoles – Steering
- Quadrupoles – focusing
- Sextupoles – chromaticity

- These components are analogous to optical elements, e.g. mirrors
  - Charged-particle optics

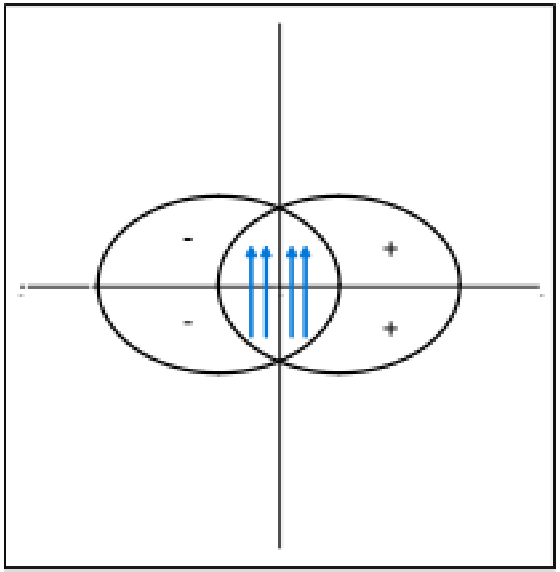
Simplest dipole: windowframe

- Assume μ=∞; field in center is uniform
- Field across coil is linear
- What happens if μ is finite?
- What happens at the ends?

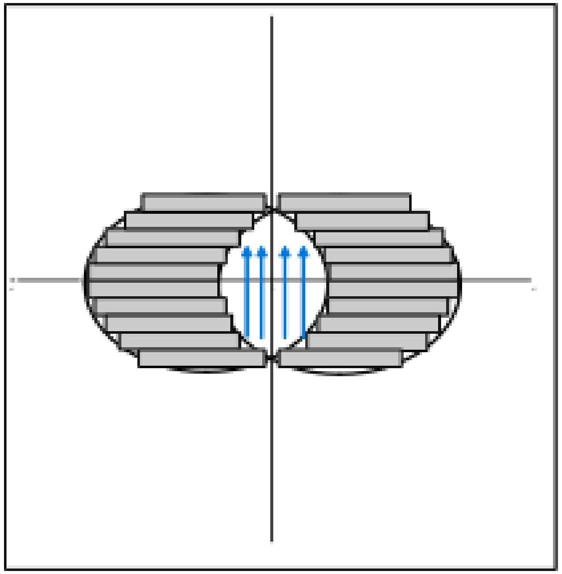


Another “simple” geometry

- Constant J in ellipse => J=0 in intersecting zone
  - Field in center is uniform => perfect dipole
  - This is the motivation for standard “Cos(θ)” dipoles



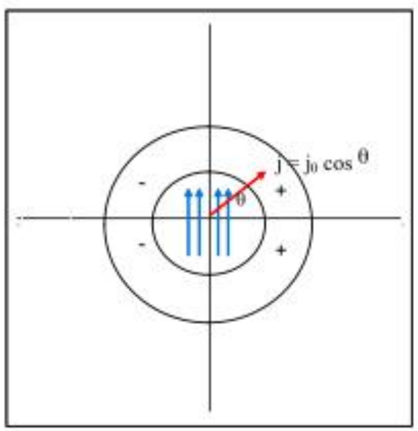
Intersecting ellipses



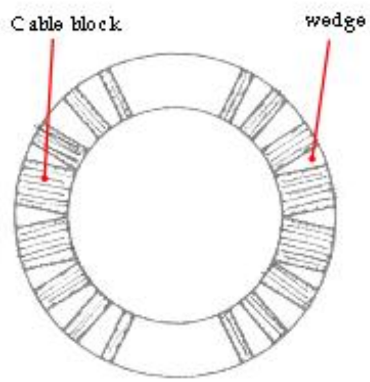
A practical (?) winding with flat cables

Transitioning from theory to practice

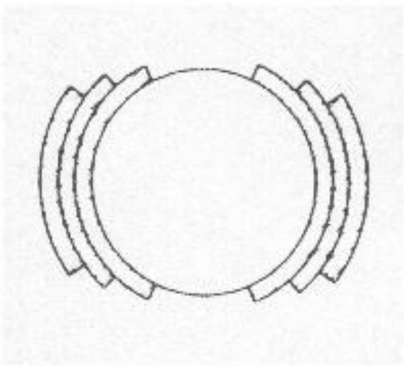
- Coil is made from a wire/cable => J~constant
  - Discretize Cos(θ) distribution using wedges
  - Ends must allow beam-passage
  - These “details” introduce errors in the form of harmonic content



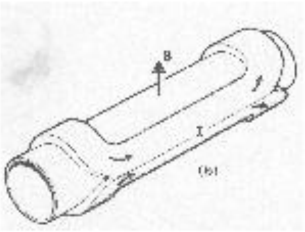
An ideal cosθ



A practical winding with one layer and wedges
[from M. N. Wilson, pg. 33]



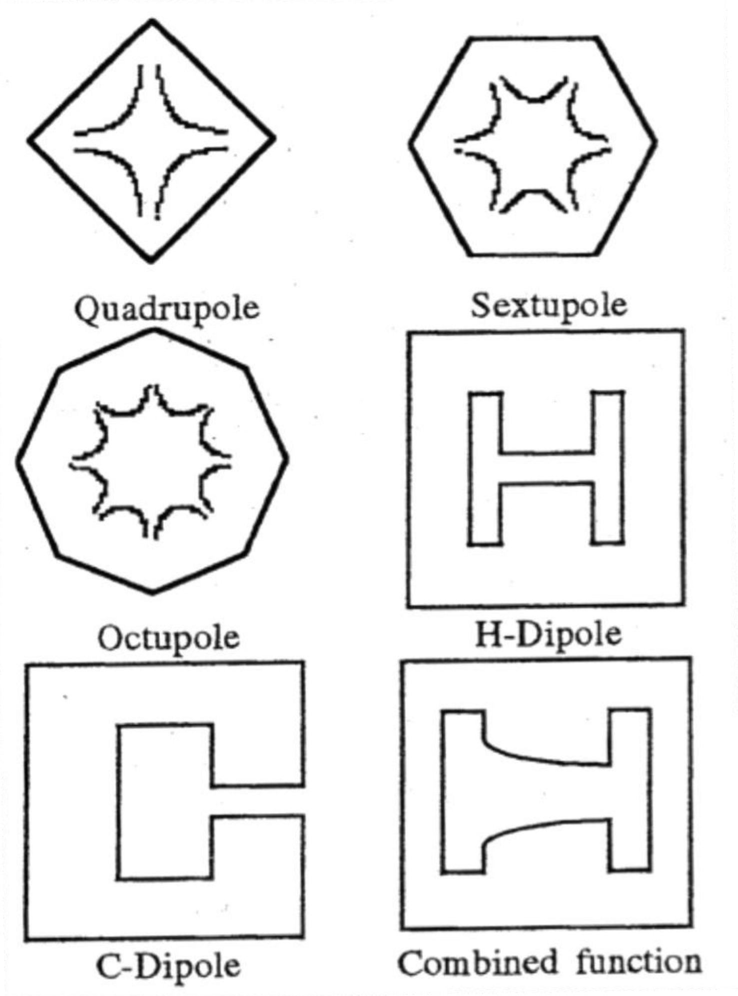
A practical winding with three layers and no wedges
[from M. N. Wilson, pg. 33]



Artist view of a cosθ magnet
[from Schmuser]

Some classic configurations

- These configurations are often mentioned in the literature
  - Combined-function magnets can take a variety of forms
    - Scalar potential can define combination of fields
    - Scalar potential can be defined for “dominant” multipole of interest – other multipoles are then added via additional energization



Review: Maxwell

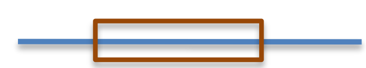
Ampere $\nabla \times \vec{H} = \vec{J}$ $\qquad \oint \vec{H} \cdot d\vec{s} = \int \vec{J} \cdot d\vec{a}$

Faraday $\nabla \times \vec{E} = \dot{\vec{B}}$ $\qquad \oint \vec{E} \cdot d\vec{s} = -\int \dot{\vec{B}} \cdot d\vec{a}$

$\nabla \cdot \vec{B} = 0$ $\qquad$ No magnetic monopoles

$$\vec{B} = \mu\mu_0\vec{H}, \qquad \mu_0 = 4\pi 10^{-7}\frac{\text{V s}}{\text{A m}}$$

Continuity across interfaces implies:



$\nabla \cdot \vec{B} = 0 \implies \Delta B_\perp = 0$ $\qquad \nabla \times \vec{H} = 0 \implies \Delta H_{||} = 0$

Allowed multipoles

$$F \equiv A + iV \tag{3}$$

It follows directly from Eqns. (2) and (3) that the complex conjugate $B^*(z)$ of the field is analytic in z and is given by:

$$B^*(z) = i\frac{dF}{dz} \tag{4}$$

It is convenient to expand the complex ~~scalar~~ potential F in a power series about a point (say $z = 0$) and analyze the 'harmonic' components:

$$F(z) = \sum_{n=1}^{\infty}\left(\frac{z}{r_p}\right)^n c_n \; ; \qquad B^*(z) = i\sum_{n=1}^{\infty}\left(\frac{z}{r_p}\right)^{n-1}\frac{nc_n}{r_p} \tag{5}$$

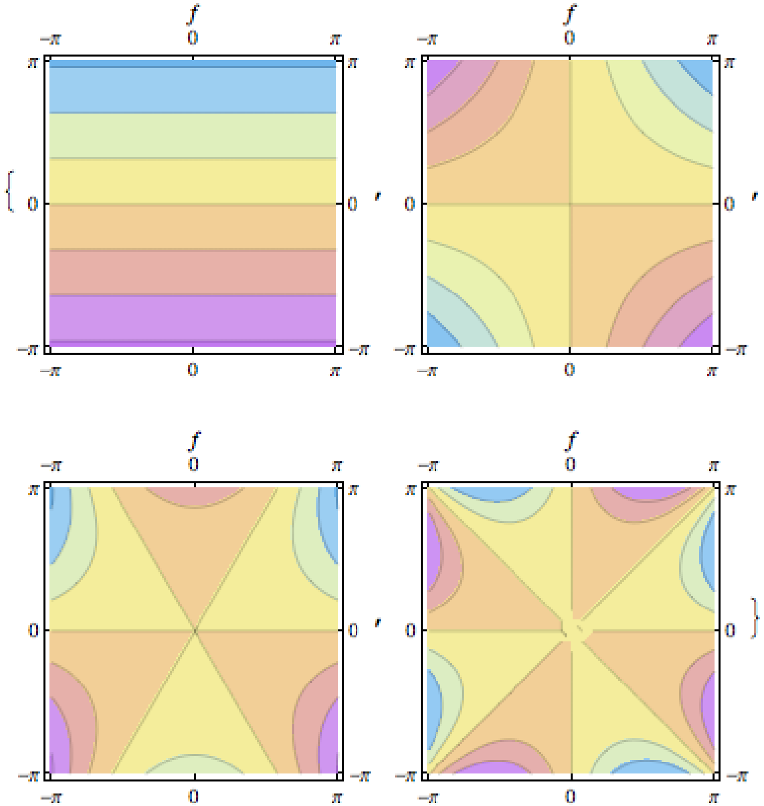
where r_p is the magnet aperture radius ($\longrightarrow$ half gap h for a dipole). For magnets exhibiting midplane symmetry, the coefficients $c_n \equiv a_n + ib_n$ are pure real (or pure imaginary if A, rather than V, is constant along the midplane). For symmetric multipole magnets (i.e. rotatable by $360°/2m$ with a change of polarity), of order m (e.g. $m = 1$ for dipole, 2 for quadrupole, etc.) the complex potential F and flux density $B^*(z)$ are

$$F(z) = \sum_{n=1}^{\infty}\left(\frac{z}{r_p}\right)^{m(2n-1)} a_{m(2n-1)} \; ; \quad B^*(z) = i\sum_{n=1}^{\infty}\left(\frac{z}{r_p}\right)^{m(2n-1)-1}\frac{m(2n-1)a_{m(2n-1)}}{r_p} \tag{6}$$

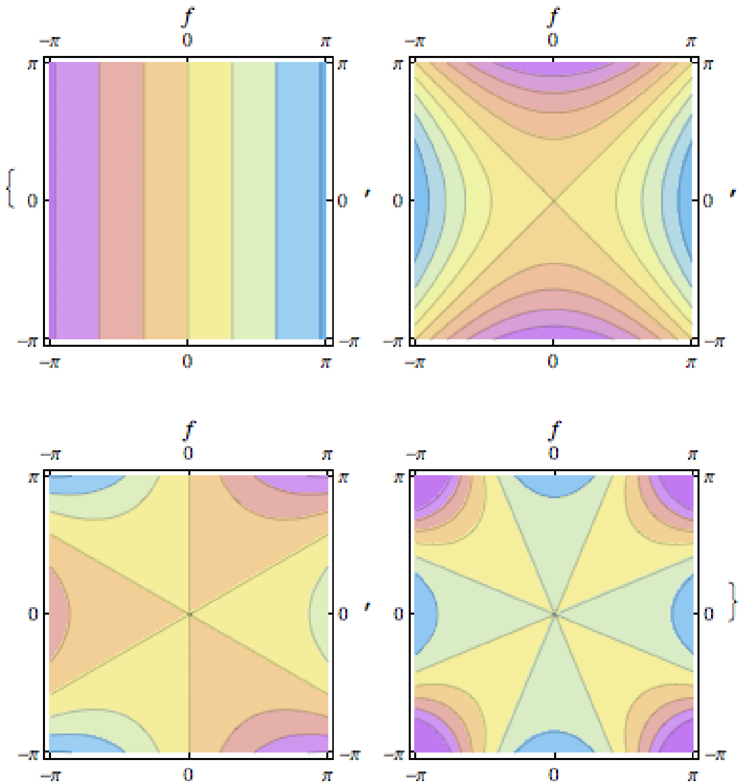
Multipole fields

- Potential isosurfaces, m=1, 2, 3, 4

Normal

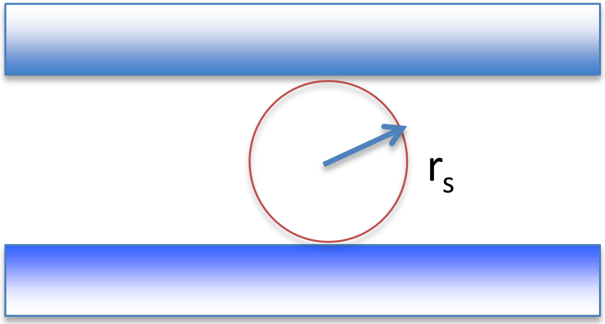


Skew



Some comments

- The series expansion is only valid out to the minimum radius r_s of any potential surface



- Non-dimensionalization by r_s is often replaced by R_{ref}, a convenient measurement radius
- The coefficients beyond B_m (the dominant mode)are then often normalized by $10^{-4}B_m$
  - resulting terms are said to be in "units"

FIELD HARMONICS OF A CURRENT LINE

- Field given by a current line (Biot-Savart law)

$$B^*(z) = \frac{\mu_0 I}{2\pi i} \frac{1}{z - z_0}$$

$$\Longrightarrow \quad F(z) = -\frac{\mu_0 I}{2\pi} Ln(z - z_0)$$

Félix Savart,
French
(June 30, 1791-March 16, 1841)

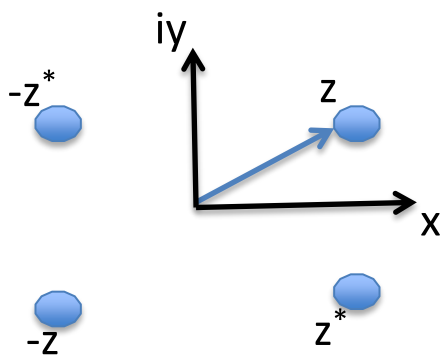
Or, in terms of multipoles:

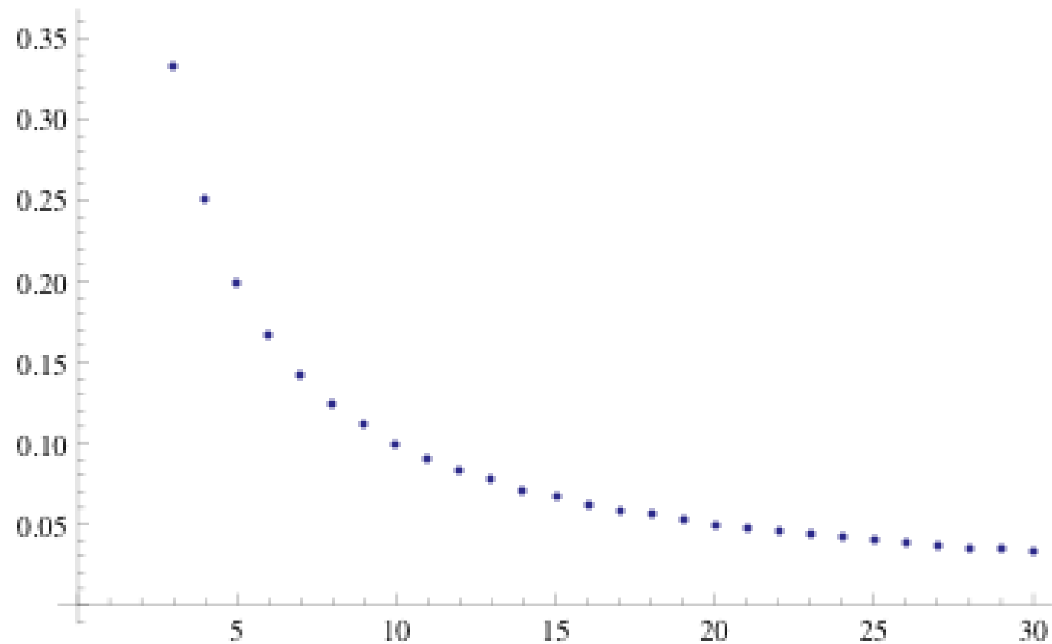
$$\begin{aligned} F(z) &= -\frac{\mu_0 I}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{z}{z_0}\right)^n \\ &= -\frac{\mu_0 I}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{R_{ref}}{z_0}\right)^n \left(\frac{z}{R_{ref}}\right)^n \end{aligned}$$

Jean-Baptiste Biot,
French
(April 21, 1774 – February 3, 1862)

FIELD HARMONICS OF A CURRENT LINE

- The multipoles of a line current then scale like 1/n
  - The details of the decay depend on the line current position
  - Adding multiple line currents judiciously positioned can result in a multipole field of order m with fairly small multipoles n≠m





The line currents can be connected so as to create a dipole, quadrupole, etc

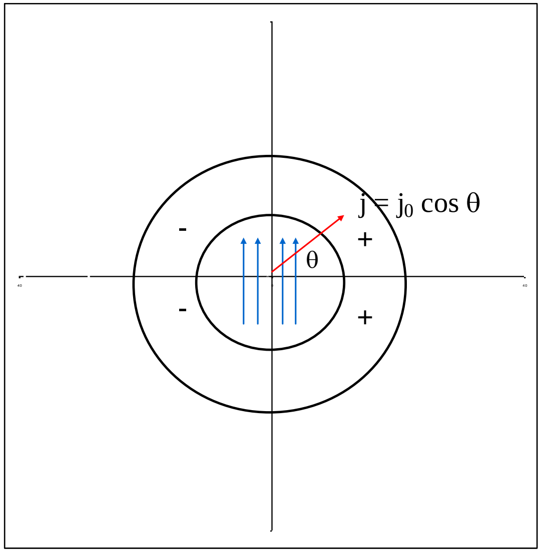
HOW TO GENERATE A PERFECT FIELD

- Perfect dipoles
  - <u>Cos theta</u>: proof – homework from last Monday

$$j(\theta) = j_0 \cos(m\theta)$$

The vector potential reads

$$A_z(\rho,\phi) = \frac{\mu_0 j}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\rho}{\rho_0}\right)^n \cos[n(\phi-\theta)]$$



and substituting one has

$$A_z(\rho,\phi) = \frac{\mu_0 j_0}{2\pi} \sum_{n=1}^{\infty} \frac{1}{n} \left(\frac{\rho}{\rho_0}\right)^n \int_0^{2\pi} \cos(m\theta)\cos[n(\phi-\theta)]d\theta$$

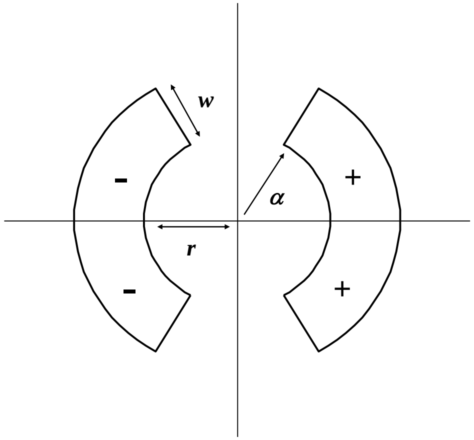
using the orthogonality of Fourier series

$$A_z(\rho,\phi) = \frac{\mu_0 j_0}{2m} \left(\frac{\rho}{\rho_0}\right)^m \cos(m\theta)$$

Basic features of "sector" coils
(Ezio Todesco)

- We compute the central field given by a sector dipole with uniform current density *j*

$$I \rightarrow j\rho d\rho d\theta$$

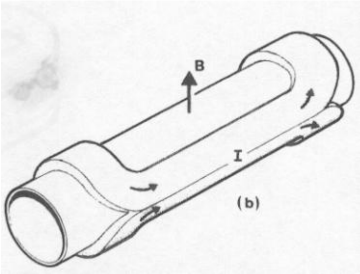


Taking into account of current signs

$$B_1 = -4\frac{j\mu_0}{2\pi}\int_0^{\alpha}\int_r^{r+w}\frac{\cos\theta}{\rho}\rho d\rho d\theta = -\frac{2j\mu_0}{\pi}w\sin\alpha$$

This simple computation is full of consequences

- $B_1 \propto$ current density (obvious)
- $B_1 \propto$ coil width w (less obvious)
- B_1 is independent of the aperture r (much less obvious)



- For a $\cos\theta$,

$$B_1 = -4\frac{j\mu_0}{2\pi}\int_0^{\pi/2}\int_r^{r+w}\frac{\cos^2\theta}{\rho}\rho d\rho d\theta = -\frac{j\mu_0}{2}w$$

SECTOR COILS FOR DIPOLES

- Multipoles of a sector coil

$$C_n = -2\frac{j\mu_0 R_{ref}^{n-1}}{2\pi}\int_{-\alpha}^{\alpha}\int_{r}^{r+w}\frac{\exp(-in\theta)}{\rho^n}\rho d\rho d\theta = -\frac{j\mu_0 R_{ref}^{n-1}}{\pi}\int_{-\alpha}^{\alpha}\exp(-in\theta)d\theta\int_{r}^{r+w}\rho^{1-n}d\rho$$

for n=2 one has

$$B_2 = -\frac{j\mu_0 R_{ref}}{\pi}\sin(2\alpha)\log\left(1+\frac{w}{r}\right)$$

and for n>2

$$B_n = -\frac{j\mu_0 R_{ref}^{n-1}}{\pi}\frac{2\sin(\alpha n)}{n}\frac{(r+w)^{2-n}-r^{2-n}}{2-n}$$

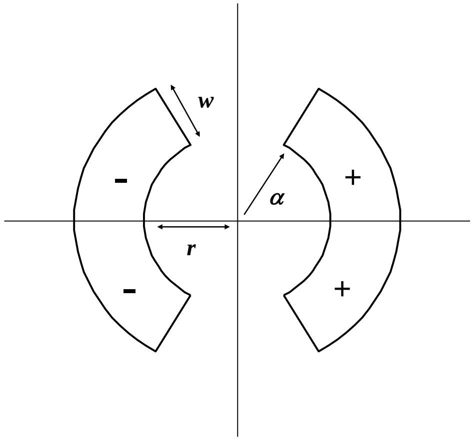
- Main features of these equations
  - Multipoles *n* are proportional to sin (*n* angle of the sector)
    - They can be made equal to zero !
  - Proportional to the inverse of sector distance to power *n*
    - High order multipoles are not affected by coil parts far from the centre

Using free parameters

- First allowed multipole B_3 (sextupole)

$$B_3 = \frac{\mu_0 jR_{ref}^2}{\pi} \frac{\sin(3\alpha)}{3} \left(\frac{1}{r} - \frac{1}{r+w} \right)$$

for $\alpha=\pi/3$ (i.e. a 60° sector coil) one has $B_3=0$



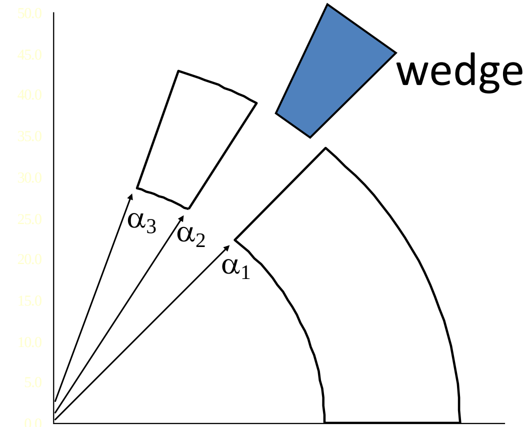
- Second allowed multipole B_5 (decapole)

$$B_5 = \frac{\mu_0 jR_{ref}^4}{\pi} \frac{\sin(5\alpha)}{5} \left(\frac{1}{r^3} - \frac{1}{(r+w)^3} \right)$$

for $\alpha=\pi/5$ (i.e. a 36° sector coil) or for $\alpha=2\pi/5$ (i.e. a 72° sector coil)

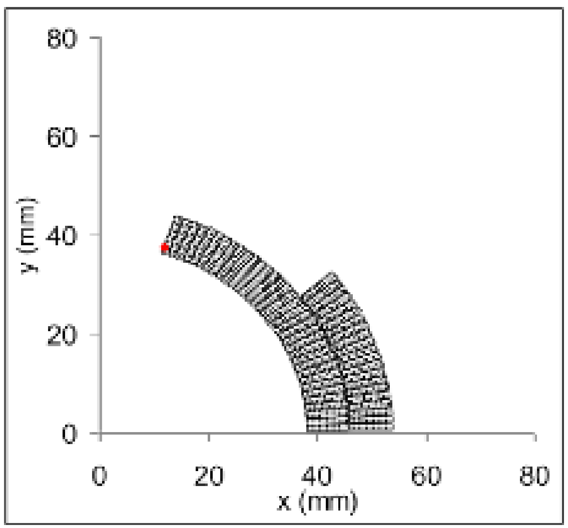
one has $B_5=0$

- With one sector one cannot set to zero both multipoles … but it can be done with more sectors!

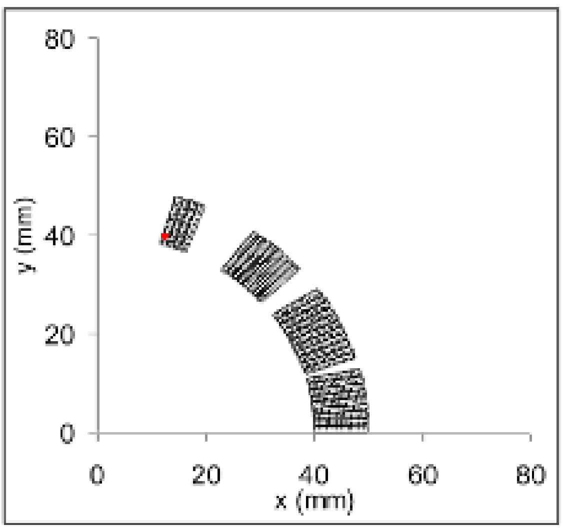


Examples of real magnets

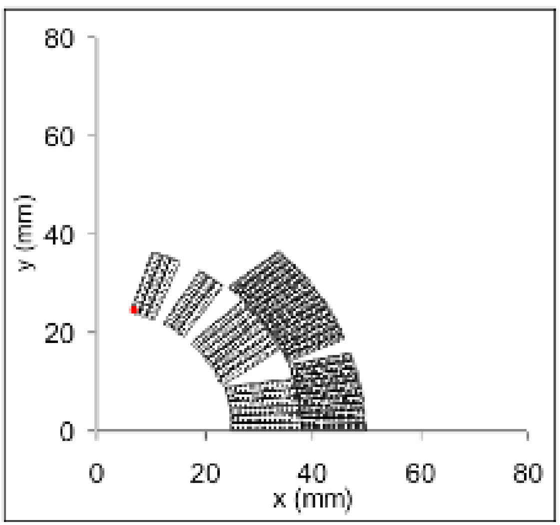
- Number of sectors is chosen based on:
  - Multipole content that can be tolerated
  - Fabrication issues



Tevatron main dipole –
location of the peak field



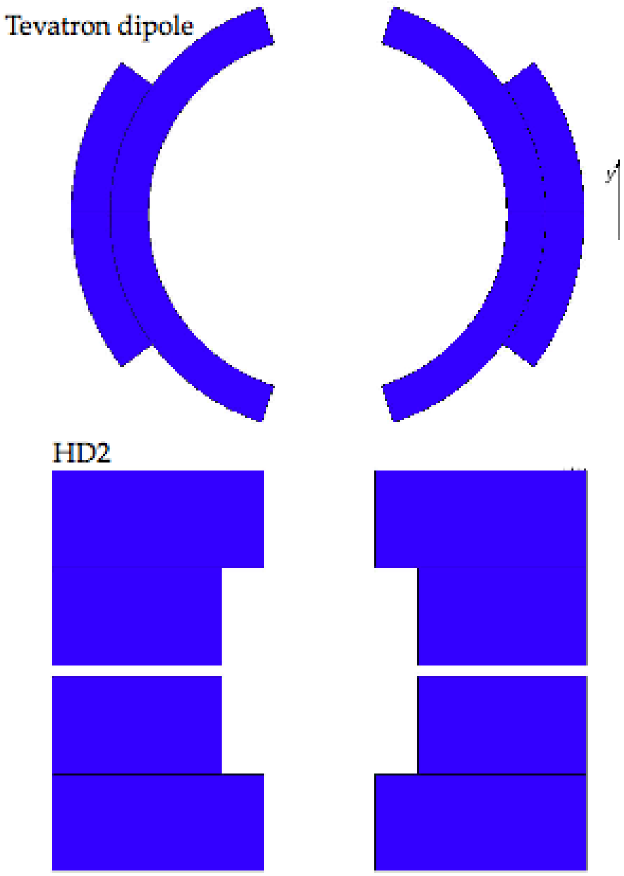
RHIC main dipole –
location of the peak field

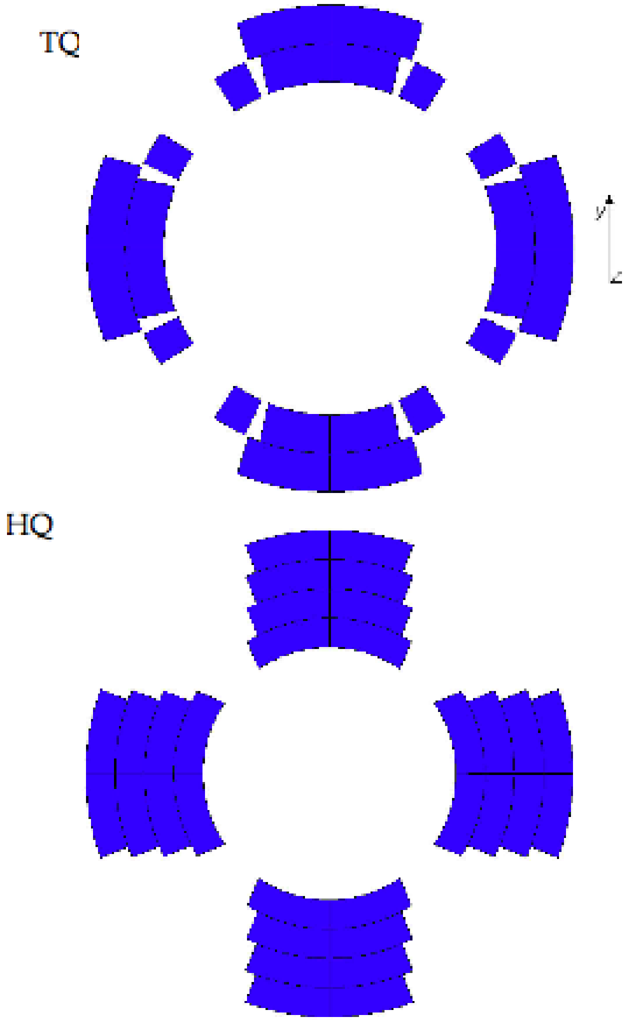


LHC main dipole –
location of the peak field

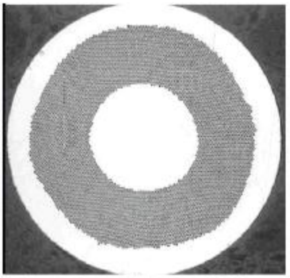
Example geometries for real superconducting accelerator magnets

Paolo Ferracin, USPAS 2009

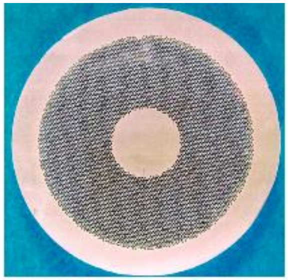




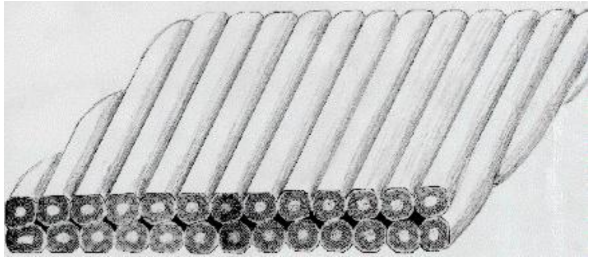
Real superconducting magnets: Basic design / fabrication

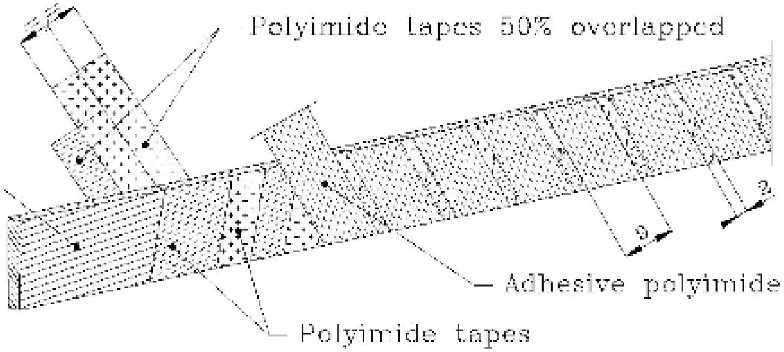


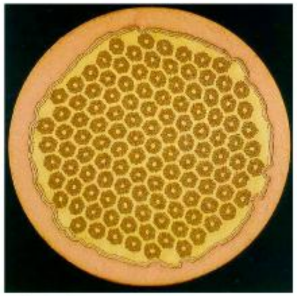
NbTi LHC wire (A. Devred, [1])



NbTi SSC wire (A. Devred, [1])

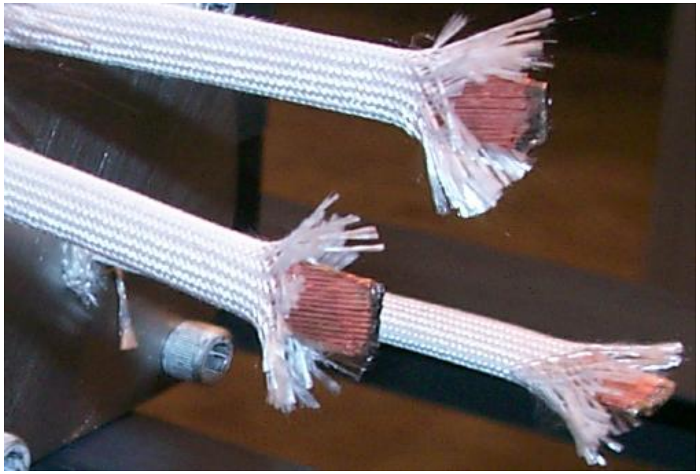






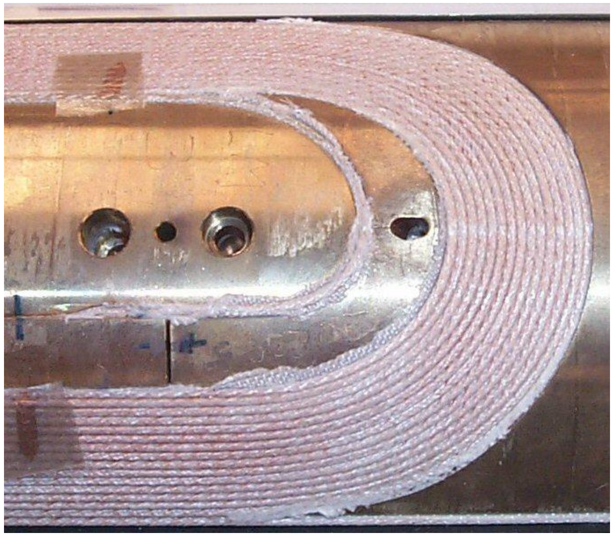
Nb_3Sn bronze-process wire (A. Devred, [1])

Nb_3Sn PIT process wire (A. Devred, [1])

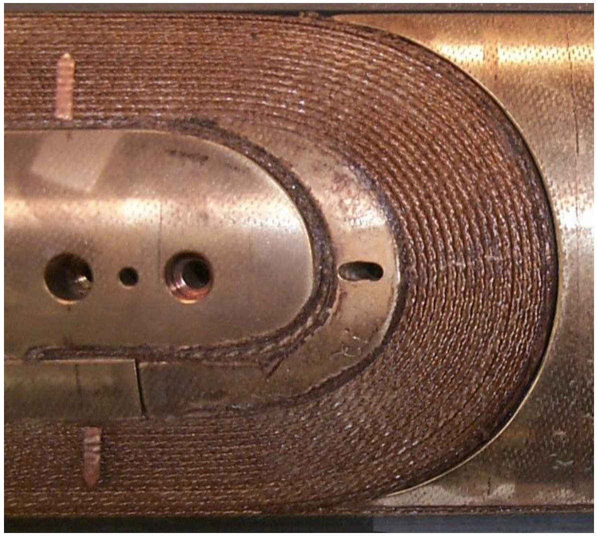


Overview of Nb_3Sn coil fabrication stages

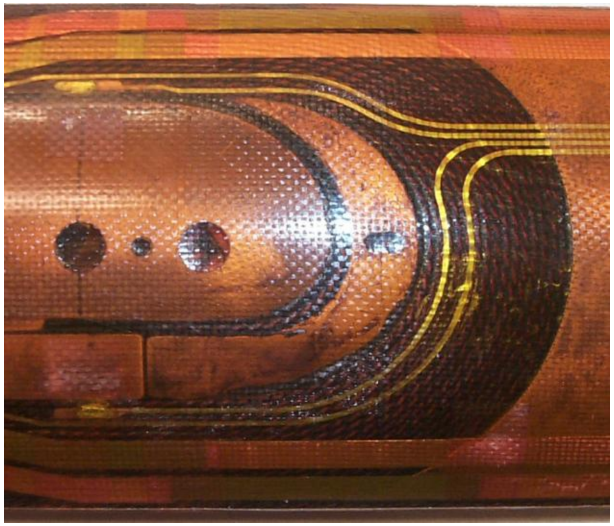
After winding



After reaction

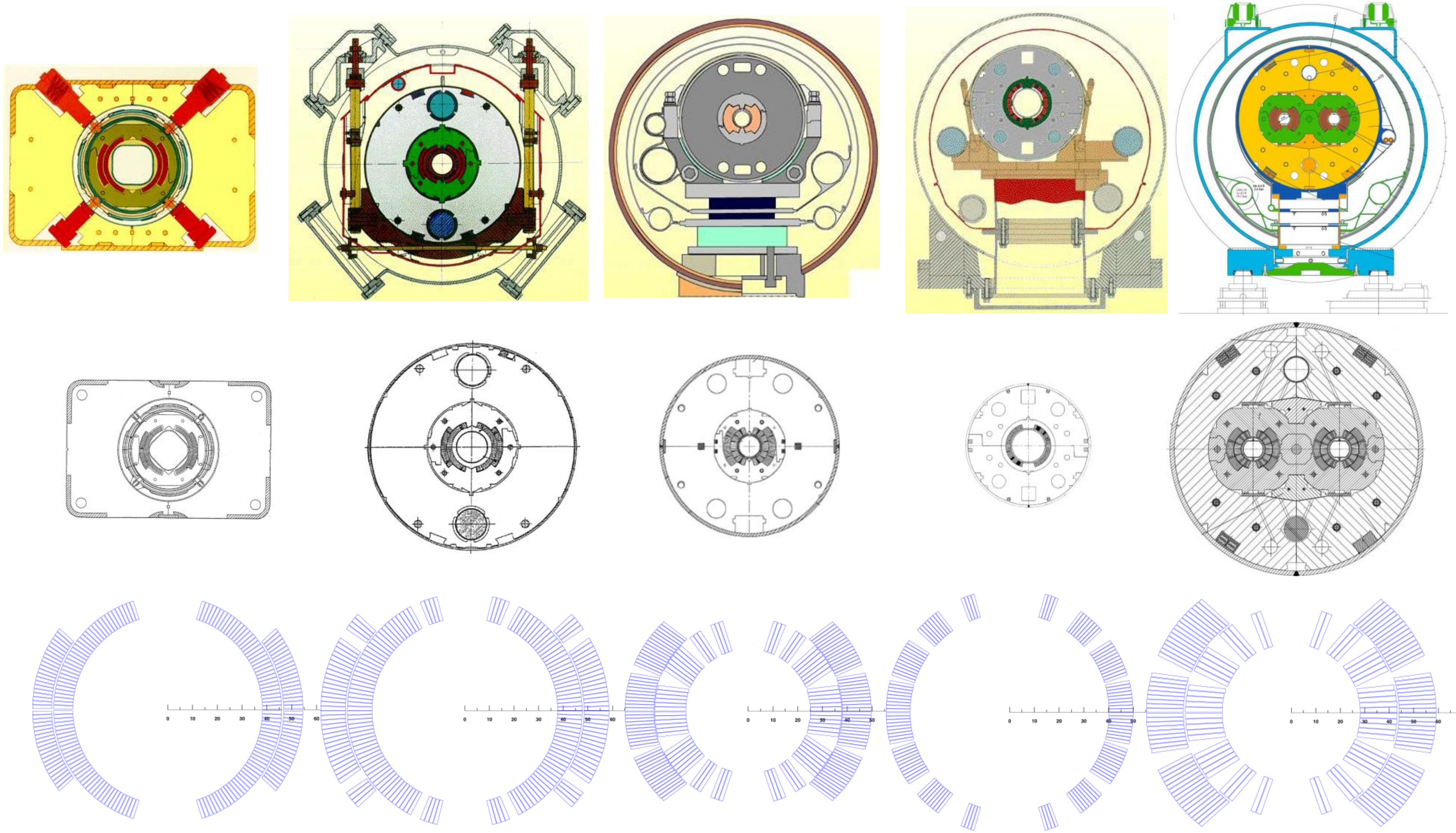


After impregnation



Overview of accelerator dipole magnets

Tevatron **HERA** **SSC** **RHIC** **LHC**

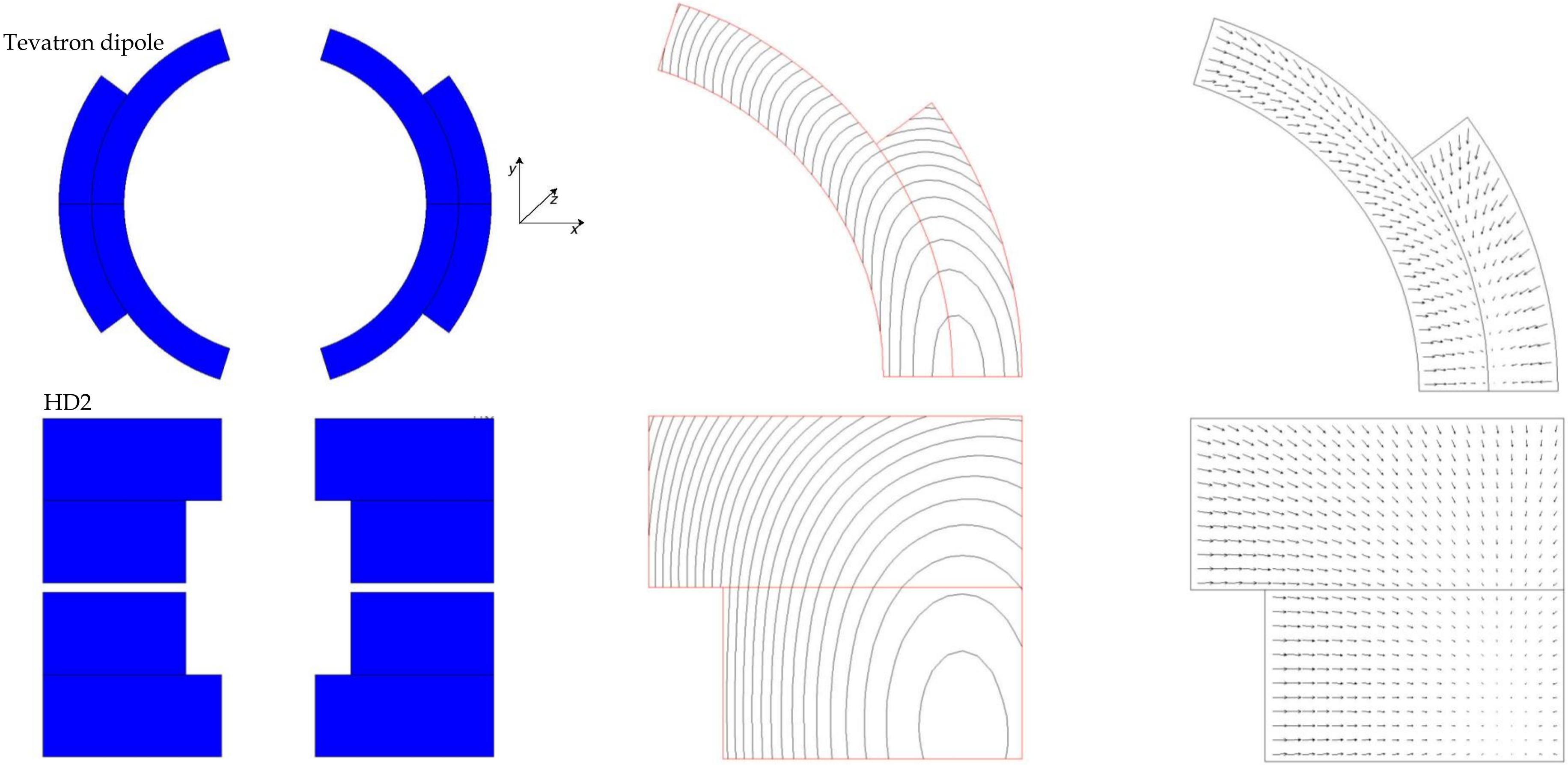


Design issues

- Superconducting magnets store energy in the magnetic field
  - Results in significant mechanical stresses via **Lorentz forces** acting on the conductors; these forces must be controlled by structures
  - **Conductor stability** concerns the ability of a conductor in a magnet to withstand small thermal disturbances, e.g. conductor motion or epoxy cracking, fluxoid motion, etc.
  - The stored energy can be extracted either in a controlled manner or through sudden loss of superconductivity, e.g. via an irreversible instability – a **quench**
    - In the case of a quench, the stored energy will be converted to heat; **magnet protection** concerns the design of the system to appropriately distribute the heat to avoid damage to the magnet

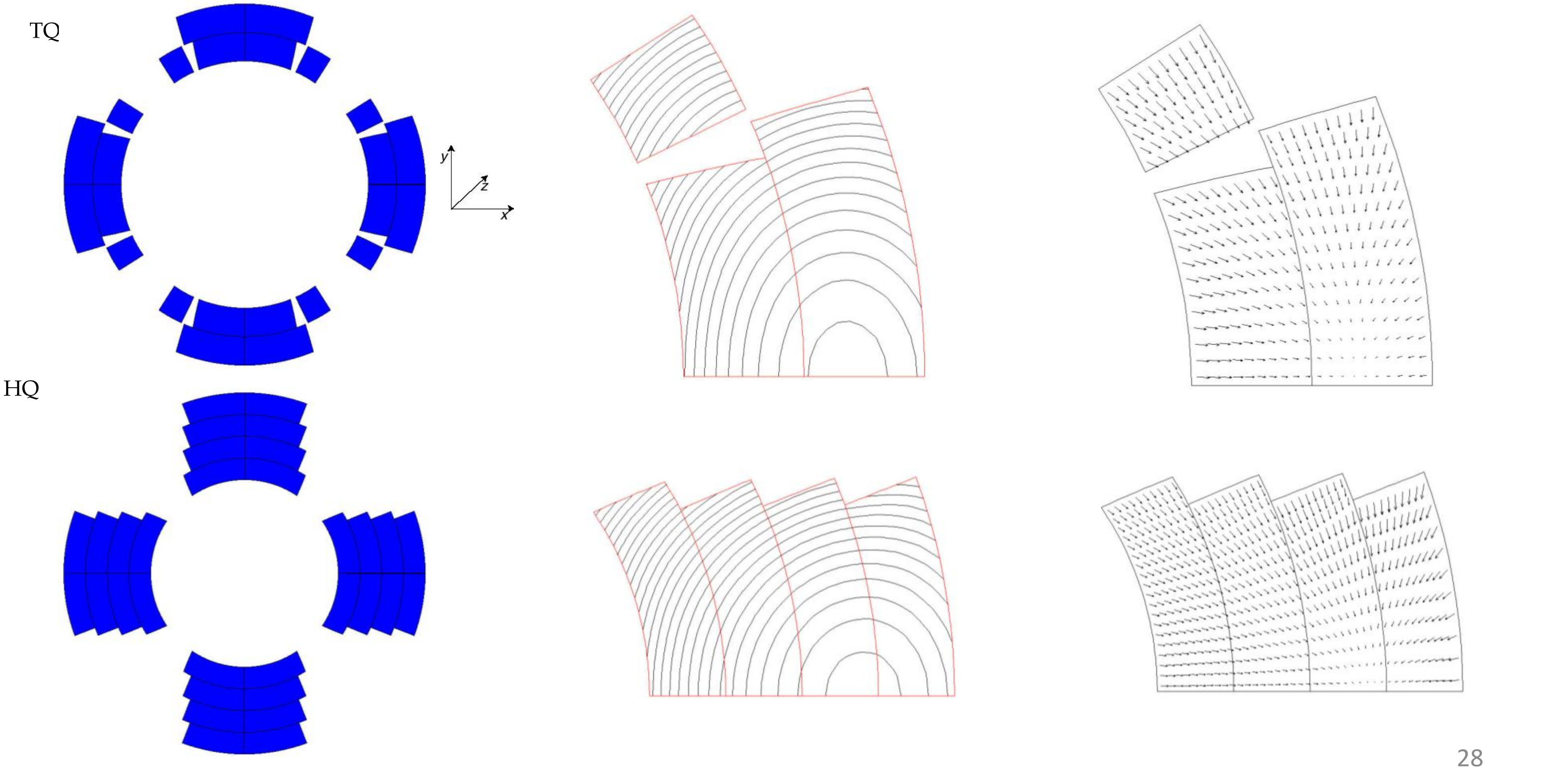
Lorentz force - Dipole magnets

- The Lorentz forces in a dipole magnet tend to push the coil
  - Towards the mid plane in the vertical-azimuthal direction (F_y, $F_\theta < 0$)
  - Outwards in the radial-horizontal direction (F_x, $F_r > 0$)



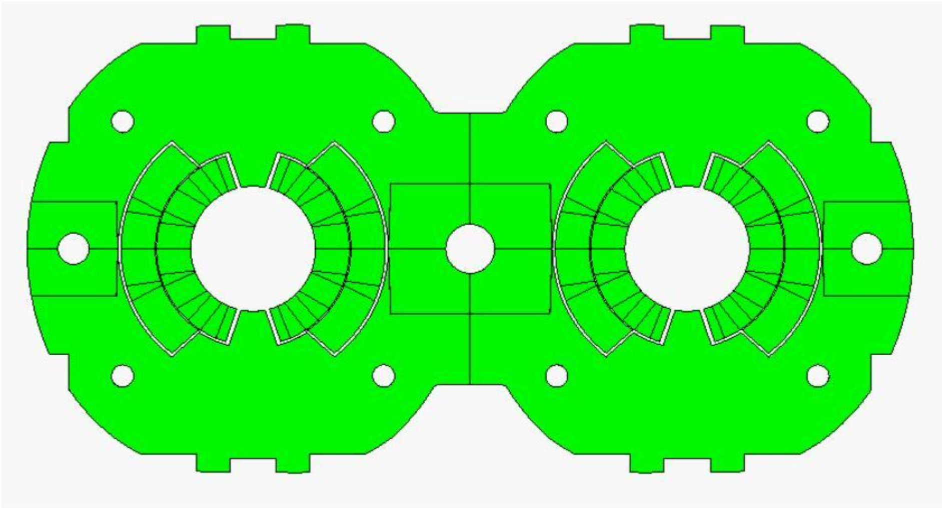
Lorentz force - Quadrupole magnets

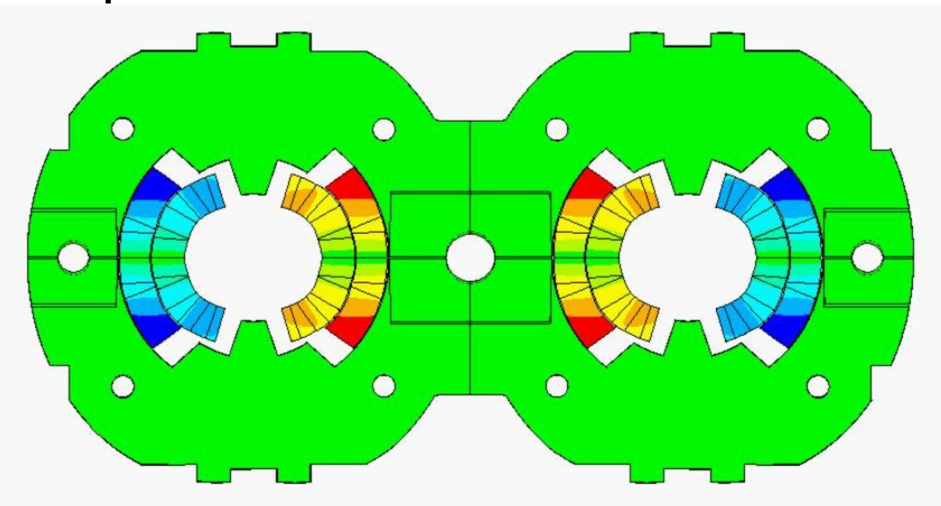
- The Lorentz forces in a quadrupole magnet tend to push the coil
  - Towards the mid plane in the vertical-azimuthal direction (F_y, $F_\theta < 0$)
  - Outwards in the radial-horizontal direction (F_x, $F_r > 0$)



Stress and strain
Mechanical design principles

LHC dipole at 0 T | LHC dipole at 9 T



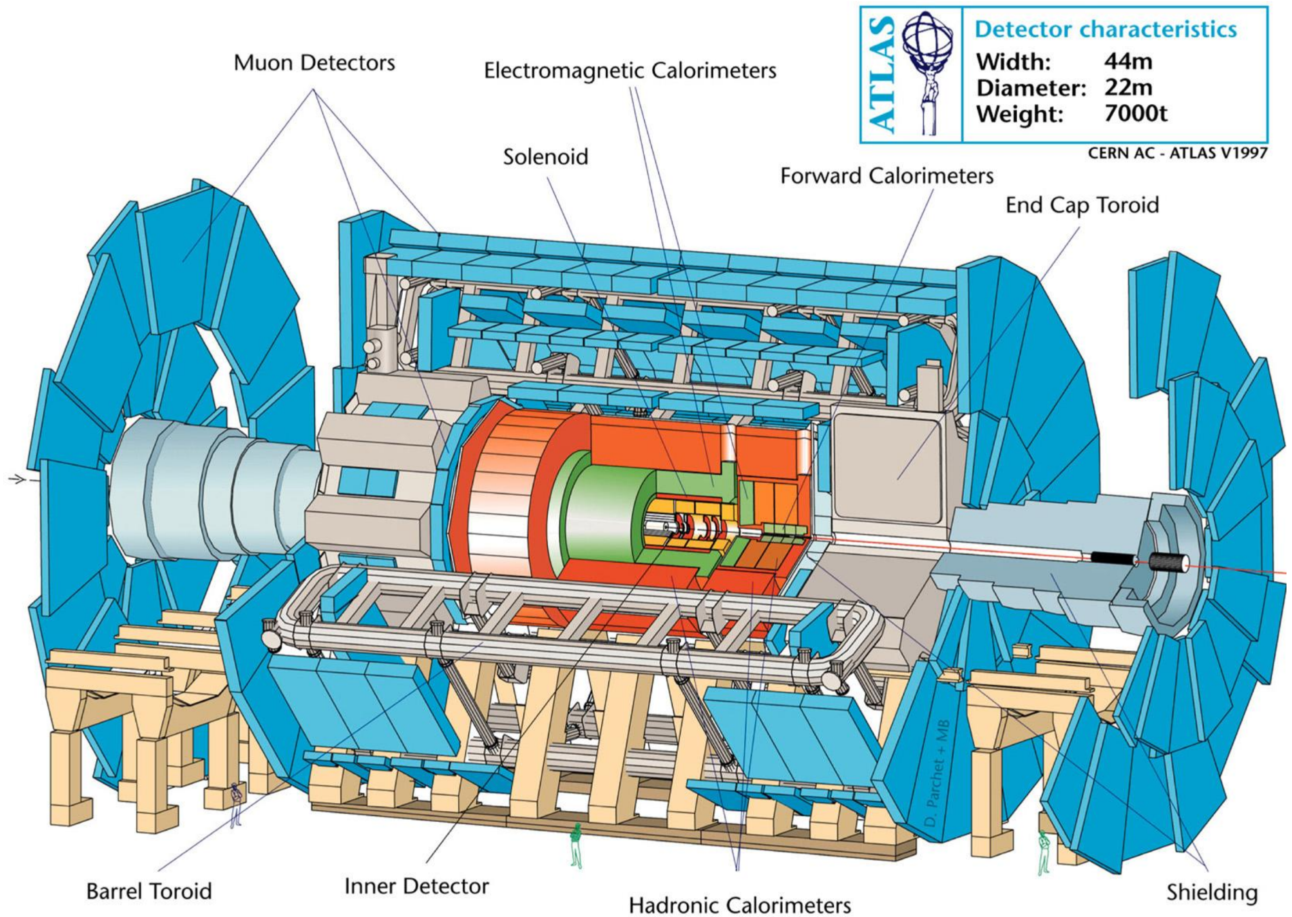


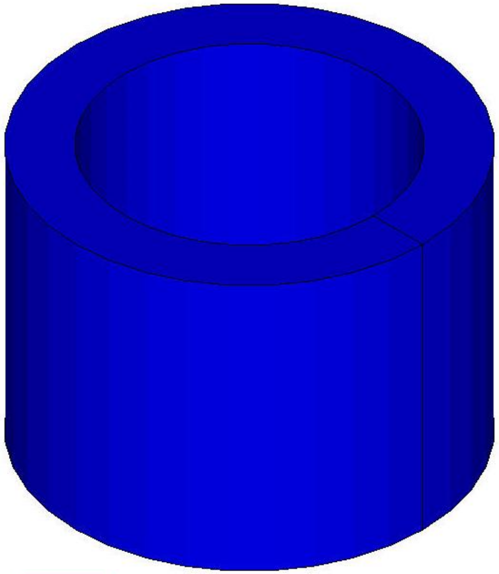
Displacement scaling = 50

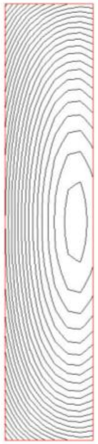
- Usually, in a dipole or quadrupole magnet, the highest stresses are reached at the mid-plane, where all the azimuthal Lorentz forces accumulate (over a small area).

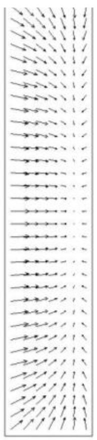
Lorentz force - Solenoids

- The Lorentz forces in a solenoid tend to push the coil
  - Outwards in the radial-direction ($F_r > 0$)
  - Towards the mid plane in the vertical direction ($F_{y,} < 0$)



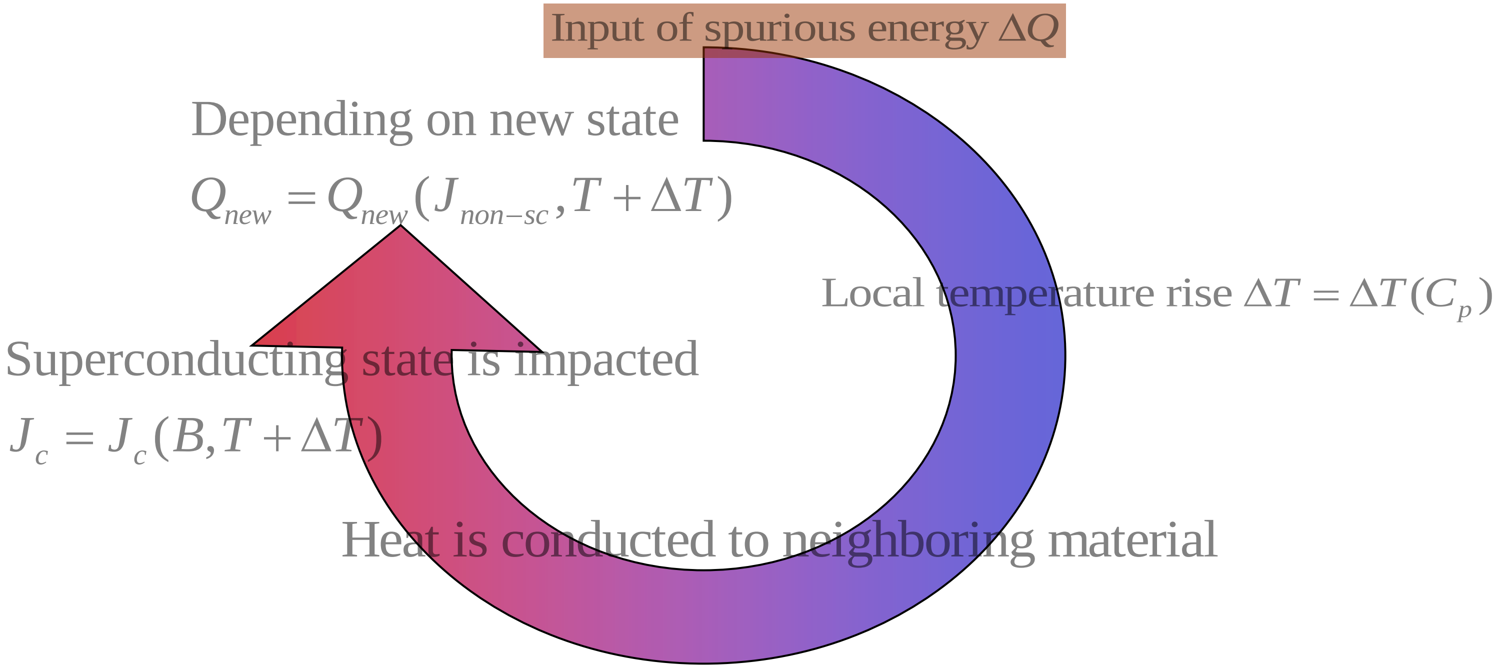






Concept of stability

- The concept of superconductor stability concerns the interplay between the following elements:
  - The addition of a (small) thermal fluctuation local in time and space
  - The heat capacities of the neighboring materials, determining the local temperature rise
  - The thermal conductivity of the materials, dictating the effective thermal response of the system
  - The critical current dependence on temperature, impacting the current flow path
  - The current path taken by the current and any additional resistive heating sources stemming from the initial disturbance



Calculation of the bifurcation point for superconductor instabilities

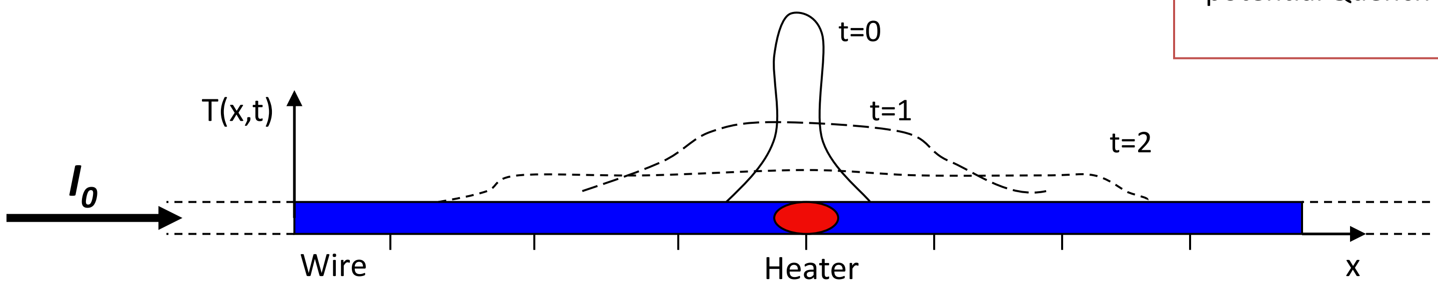
Thanks to Matteo Allesandrini, Texas Center for Superconductivity, for these calculations and slides

Heat Balance Equation in 1D, without coolant: *[W/m³]*

$$\frac{d}{dx}\left(k(T)\cdot\frac{dT}{dx}\right)+\rho(T)\cdot J^2+Q_{initial\_pulse}-C(T)_{volume}\cdot\frac{dT}{dt}=0$$

Heat conduction | Joule effect | Quench trigger | Heat stored in the material

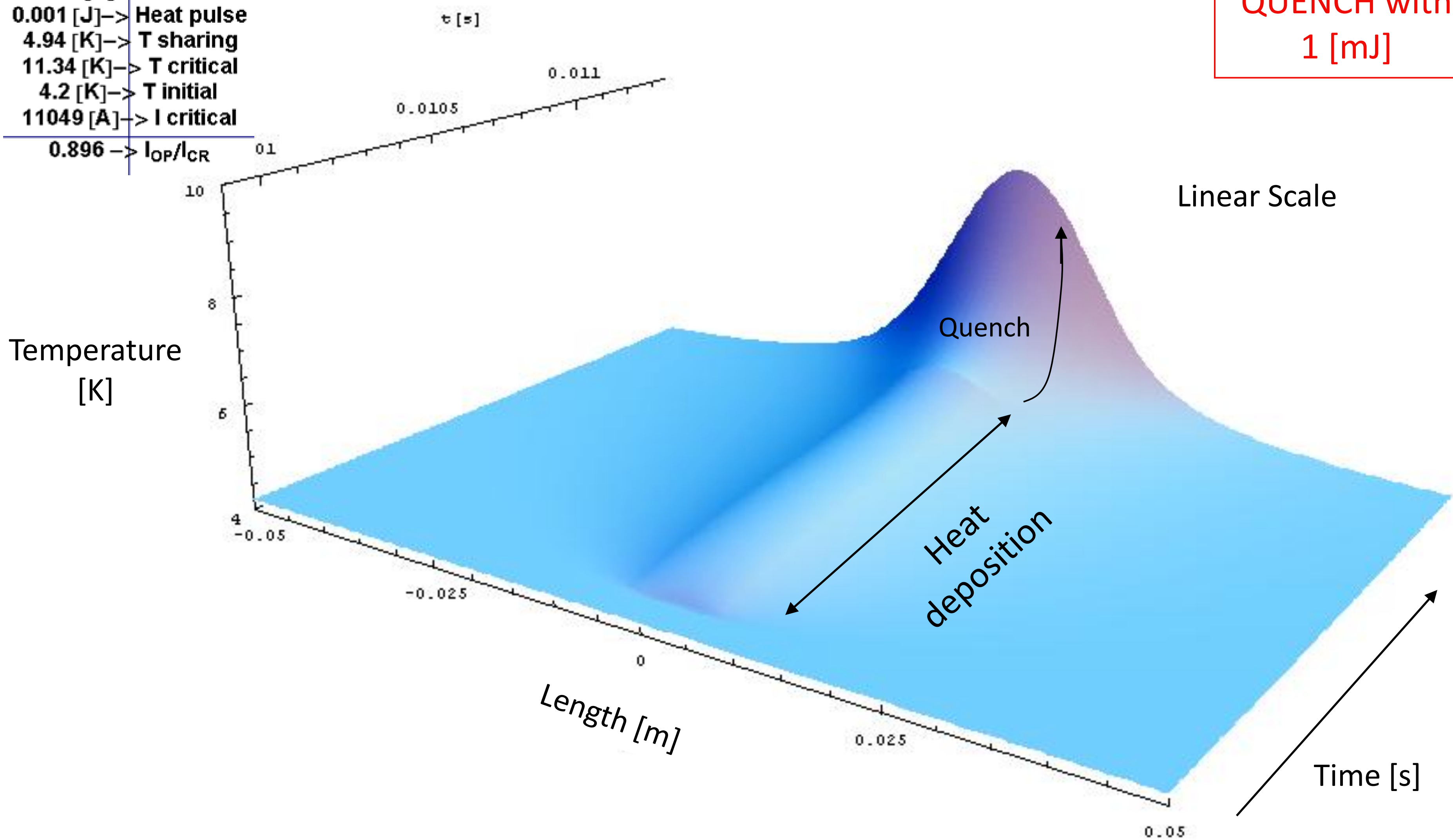
Ex. RECOVERY of a potential Quench



Example of quench initiation

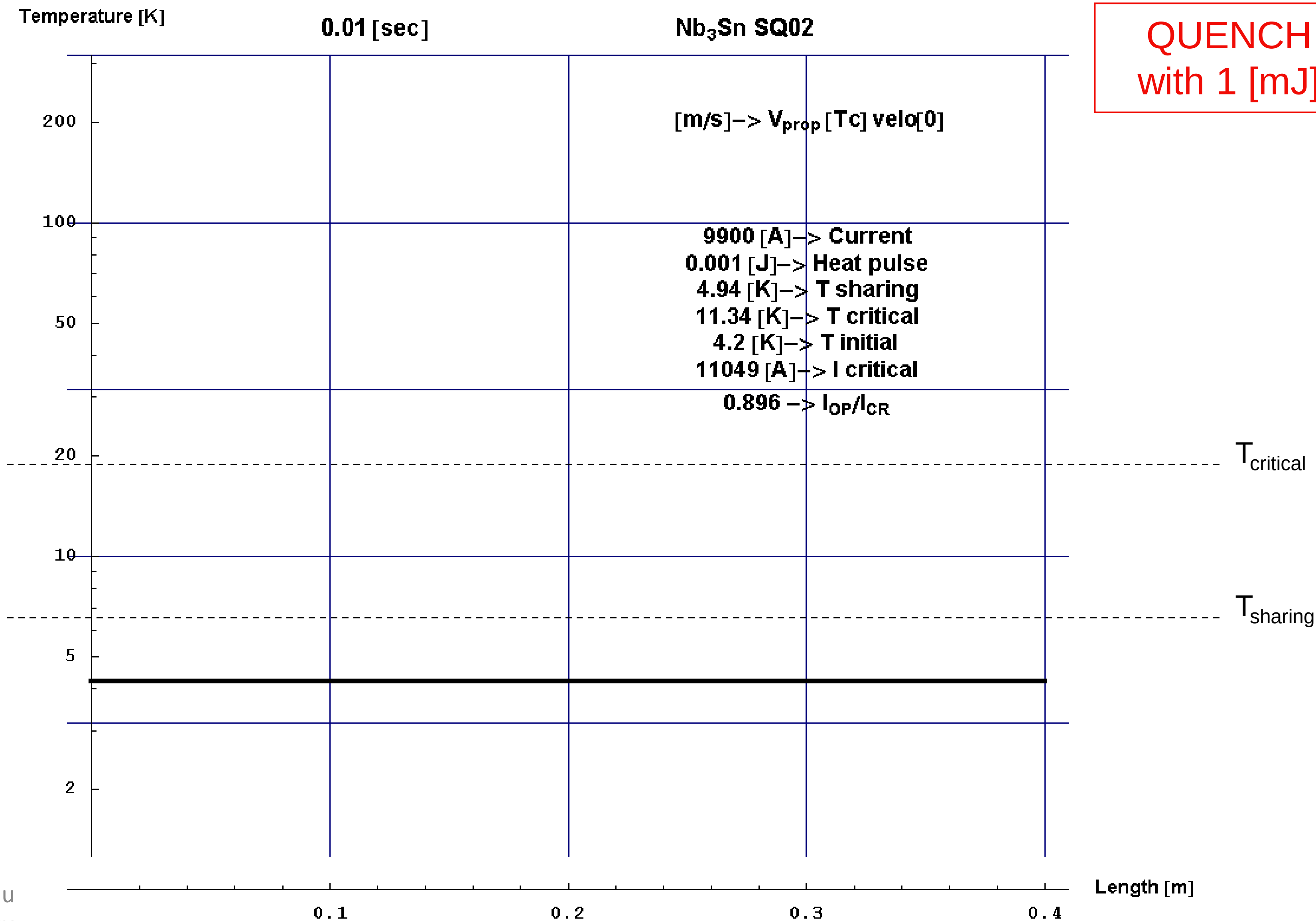
9900 [A]	–> Current
0.001 [J]	–> Heat pulse
4.94 [K]	–> T sharing
11.34 [K]	–> T critical
4.2 [K]	–> T initial
11049 [A]	–> I critical
0.896	–> I_{OP}/I_{CR}

QUENCH with 1 [mJ]



Analysis of SQ02 – quench propagation

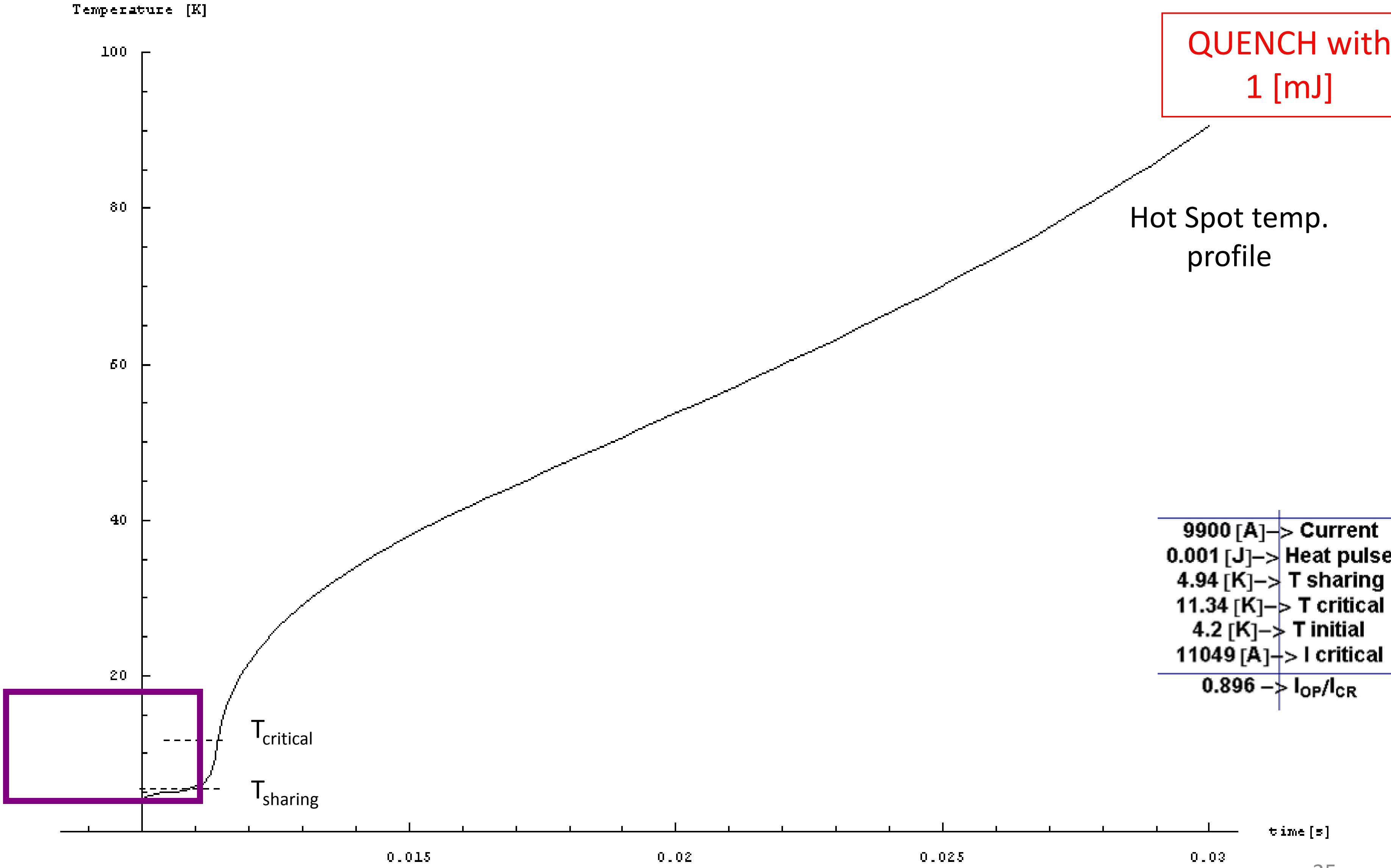
QUENCH with 1 [mJ]



Flux Ju
Superc

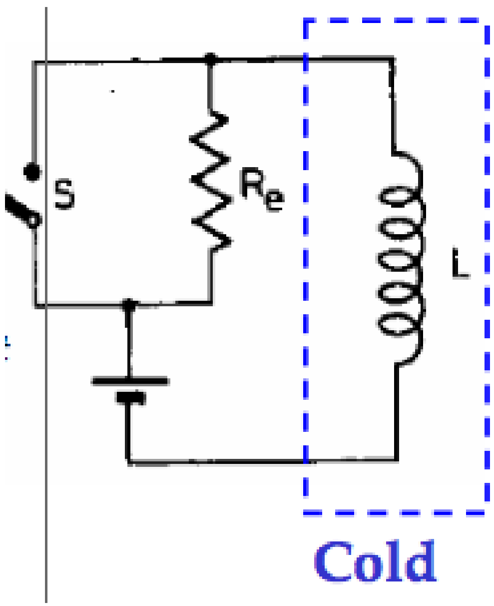
Analysis of SQ02 – quench propagation

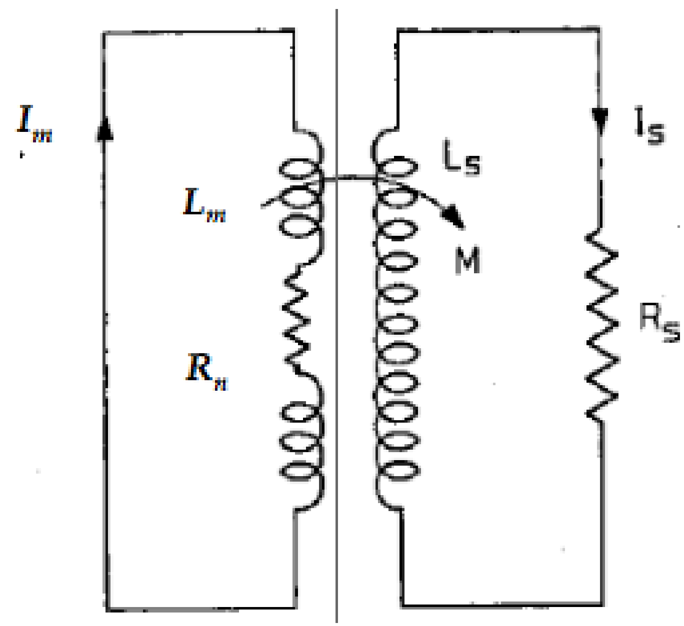
QUENCH with 1 [mJ]



Magnet protection

- The quench propagation aids in distributing stored energy to the rest of the magnet
- Often we accelerate the process by actively heating the magnet once a quench initiation has been detected ("Active protection")
- If possible, much of the energy is also absorbed by a dump resistor
- The energy can also be absorbed by inductive coupling to a secondary





Permeability and field-lines

Problem 1: find the functional relationship $\alpha 1=f(\mu 1,\mu 2,\alpha 2)$. Plot the function for $\mu 1=1$, $\mu 2=1$ and $\mu 2=10$